



GS-324

VI Semester B.A./B.Sc. Examination, May/June - 2019

MATHEMATICS

Mathematics - VIII

(CBCS) (Fresh+Repeaters) (2016-17 & Onwards)

Time : 3 Hours

Max. Marks : 70

Instruction : Answer **all** questions.

PART - A

1. Answer **any five** questions.

5x2=10

- Evaluate $\lim_{z \rightarrow 1+i} (z^2 + 2z)$.
- Show that $\left| \frac{z-2}{z+2} \right| = 3$ represents a circle.
- Show that $u = e^x \sin y + x^2 - y^2$ is harmonic.
- Define cross ratio of four points.
- State Liouville's Theorem.
- Evaluate $\oint (\bar{z})^2 dz$ around the circle $|z| = 1$
- Write Euler modified formula.
- State Runge- Kutta Method of order 4.

PART- B

Answer **four** full Questions.

4x10=40

2. (a) Show that $\arg \left(\frac{z-1+i}{z+1} \right) = \frac{\pi}{4}$

(b) Prove that $\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}$ and $\frac{\partial u}{\partial \theta} = -\frac{\partial v}{\partial r}$

OR

3. (a) Prove that $\lim_{z \rightarrow 0} \left(\frac{\bar{z}}{z} \right)$ does not exist.

(b) Show that $f(z) = \sin z$ is analytic and hence prove that $f'(z) = \cos z$.

4. (a) Show that $u = y^3 - 3x^2y$ is harmonic and find its harmonic conjugate.

(b) If $f(z) = u + iv$ is an analytic function then prove that the curves $u(x,y) = c_1$, $v(x,y) = c_2$ form two orthogonal families.

OR

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5. (a) Find the analytic function $f(z) = u + iv$ given that $u - v = e^x (\cos y - \sin y)$.
- (b) If $f(z) = u + iv$ is analytic then show that $\left[\frac{\partial}{\partial x} |f(z)| \right]^2 + \left[\frac{\partial}{\partial y} |f(z)| \right]^2 = |f'(z)|^2$
6. (a) Evaluate $\int_{(0, 3)}^{(2, 4)} [(2y + x^2)dx + (3x - y)dy]$ using the substitution $x = 2t, y = t^2 + 3$.
- (b) State and Prove Cauchy's Integral Formula.

OR

7. (a) Evaluate $\oint_c \frac{z - 4}{z(z^2 + 9)} dz$ where c is the circle $|z| = 1$.
- (b) State and prove Cauchy's Integral Theorem.
8. (a) Prove that the Bilinear Transformation preserves the cross ratio of four points.
- (b) Discuss the transformation $w = z^2$.

OR

9. (a) Find the Bilinear Transformation which maps $z = 1, e^i, -1$ on to $w = i, 0, -i$.
- (b) Show that the transformation $W = 1/z$ transforms a circle to a circle or to a straight line.

PART- C

Answer **two** full questions.

2x10=20

10. (a) Find the root of the equation $x^3 - 4x + 1 = 0$ by Regula-falsi method upto three decimal places.
- (b) Use Newton-Raphson Method to find a real root of the equation. $x^3 - 9x + 1 = 0$ near $x = 3$.

OR

11. (a) Solve by Gauss- Jacobi method.
- $$\begin{aligned} 10x + 2y + z &= 9 \\ x + 10y - z &= -22 \\ -2x + 3y + 10z &= 22 \end{aligned}$$
- (b) Find the largest Eigen Value of the matrix.

$$A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$



12. (a) Use Taylor's series method to find y at $x=0.1$ considering terms upto the third degree given $\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 1$.
- (b) Using Euler's modified method, find $y(0.2)$ given $\frac{dy}{dx} = x - y^2$, $y(0) = 1$ taking $h=0.1$.

OR

13. (a) Use Euler's Method to solve $\frac{dy}{dx} = x + y$, $y(0) = 1$ for $x=0.0$ (0.2) 1.0
- (b) Using Runge-Kutta Method find $y(0.2)$ for $\frac{dy}{dx} = x + y$; $y(0) = 1$ taking $h=0.2$.

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- (b) If $f(z) = u + iv$ is analytic then show that $\left[\frac{\partial}{\partial x} |f(z)| \right]^2 + \left[\frac{\partial}{\partial y} |f(z)| \right]^2 = |f'(z)|^2$
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OR

11. (a) Solve by Guass- Jacobi method.
 $10x + 2y + z = 9$
 $x + 10y - z = -22$
 $-2x + 3y + 10z = 22$
- (b) Find the largest Eigen Value of the matrix.

$$A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$